

Some New Form Of Nano Locally Closed Sets In Nano Topological Spaces

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Abstract

The aim of this paper, we introduce $N\lambda\psi g$ -locally closed set, $N\lambda\Psi g$ -locally closed continuous functions and $N\lambda\Psi g$ - locally closed irresolute functions nano topological spaces and some relationship between them.

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1 Introduction

M.K.R.S.Veerakumar [6] was introduced the notion of Ψ closed sets in topological spaces. Maki [3] introduced the notion of Λ -sets in topological spaces in 1986. Λ -set is a set A which is equal to its kernel, i.e., to the intersection of all open supersets of A . Lellis Thivagar introduced [2] nano topological space with respect to a subset X of a universe which is defined in terms of lower and upper approximation of X . The elements of nano topological space are called nano open sets. He has also defined nano closed sets, nano interior and nano closure of a set. He also introduced the weak forms of nano open sets. P.Subbulakshmi and N.R.Santhi Maheswari [5] nano $N\Lambda_{\Psi}(A)$ sets, nano $N\Lambda_{\Psi}^*(A)$ sets, nano Λ_{Ψ} -set and nano Λ_{Ψ}^* -set in nano topological spaces and investigate some of their properties and we defined [7] $N(\Lambda, \Psi)$ -closed sets, $N(\Lambda, \Psi)$ -open sets and $N\lambda\Psi g$ -closed sets. Moreover we introduced [8] $N\lambda\Psi g$ -continuous functions and $N\lambda\Psi g$ -irresolute functions in nano topological spaces. In this paper, we discuss about the concept of $N\lambda\psi g$ -locally closed sets in nano topological spaces. Also we describe $N\lambda\Psi g$ – locally closed continuous

function and $N\lambda\Psi g$ -locally closed irresolute function in nano topological spaces. We study some of its characterisation and discuss their relationship between them.

2 Preliminaries

Definition 2.1. [3] Let U be the Universe and R be an equivalence relation U and $\tau_R(X) = \{U, \varphi, L_R(X), U_R(X), B_R(X)\}$ where $X \subseteq U$. $\tau_R(X)$ satisfies the following axioms:

- (1) U and $\varphi \in \tau_R(X)$.
- (2) The union of elements of any subcollection of $\tau_R(X)$ is in $\tau_R(X)$.
- (3) The intersection of the elements of any finite subcollection of $\tau_R(X)$ is in $\tau_R(X)$.

We call $(U, \tau_R(X))$ is a nano topological space. The elements of $\tau_R(X)$ are called a open sets and the complement of a nano open set is called nano closed sets.

Throughout this paper $(U, \tau_R(X))$ is a nano topological space with respect to X where $X \subseteq U$, R is an equivalence relation on U , U/R denotes the family of equivalence classes of U by R .

Definition 2.2. [3] If $(U, \tau_R(X))$ is a nano topological space with respect to X . Where $X \subseteq G$ and if $A \subseteq G$, then

- The nano interior of the set A is defined as the union of all nano open subsets contained in A and is denoted by $Nint(A)$, $Nint(A)$ is the largest nano open subset of A .
- The nano closure of the set A is defined as the intersection of all nano closed sets containing A and is denoted by $Ncl(A)$. $Ncl(A)$ is the smallest nano closed set containing A .

Definition 2.3. [3] Let $(U, \tau_R(X))$ be a nano topological space and $A \subseteq G$. Then A is said to be

- (i) Nano semi-open if $A \subseteq Ncl(Nint(A))$
- (ii) Nano semi-closed if $Nint(Ncl(A)) \subseteq A$

Definition 2.4. [6] Let A be a subset of a nano topological space $(U, \tau_R(X))$. A subset $N\Lambda_\Psi(A)$ is defined as $N\Lambda_\Psi = \cap \{H/A \subseteq H \text{ and } H \in N\Psi O(U, \tau_R(X))\}$.

Definition 2.5. [6] A subset A of a nano topological space $(U, \tau_R(X))$ is called a $N\Lambda_\Psi(A)$ -set if $A = N\Lambda_\Psi(A)$. The set of all $N\Lambda_\Psi(A)$ -sets is denoted by $N\Lambda_\Psi(U, \tau_R(X))$.

Definition 2.6. [7] Let A be a subset of a nano topological space $(U, \tau_R(X))$. A subset $N(\Lambda, \Psi)$ closed if $A = B \cap C$, where B is $N\Lambda_\Psi$ set and C is a $N\Psi$ closed set. The family of all $N(\Lambda, \Psi)$ -closed sets is denoted by $N\Lambda_\Psi C(U, \tau_R(X))$.

Definition 2.7. Let $(U, \tau_R(X))$ be a nano topological space and $A \subseteq G$. Then A is said to be

1. Nano generalized closed (briefly Ng) [1] if $Ncl(A) \subseteq G$ whenever $A \subseteq G$ and G is nano-open in U .

2. Nano semi generalized closed (briefly Nsg) [1] if $Nscl(A) \subseteq G$ whenever $A \subseteq G$ and G is nano-semi open in U .
3. Nano ψ -closed [9] (briefly $N\psi$ -closed) if $Nscl(A) \subseteq G$ whenever $A \subseteq G$ and G is Nsg-open in U .
4. Nano $\lambda\psi$ generalized closed [7] (briefly $N\lambda\psi g$ -closed) if $N\psi cl(A) \subseteq G$ whenever $A \subseteq G$ and G is $N(\Lambda, \psi)$ open in U .

Theorem 2.8. [7] Let $(U, \tau_R(X))$ be a nano topological spaces. Then

1. Every nano closed set is $N\lambda\psi g$ -closed.
2. Every nano open set is $N\lambda\psi g$ -open.

Theorem 2.9 [7]

1. Arbitrary intersection of $N\lambda\psi g$ - closed sets is $N\lambda\psi g$ - closed.
2. Arbitrary union of $N\lambda\psi g$ -open sets is $N\lambda\psi g$ -open.

Definition 2.10. [2] A subset S of a topological space $(U, \tau_R(X))$ is called nano locally closed (briefly NLC) set if $S = G \cap H$ where G is nano open and H is nano closed.

Definition 2.11. [2] A function $f : (U, \tau_R(X)) \rightarrow (V, \tau'_R(Y))$ is said be NLC- continuous if $f^{-1}(G)$ is NLC set in $(U, \tau_R(X))$ for each closed set G of $(V, \tau'_R(Y))$.

Definition 2.12. [2] A function $f : (U, \tau_R(X)) \rightarrow (V, \tau'_R(Y))$ is said be NLC- irresolute if $f^{-1}(G)$ is NLC set in $(U, \tau_R(X))$ for each NLC set G of $(V, \tau'_R(Y))$.

3 $N\lambda\psi g$ - Locally closed sets

Definition 3.1. A subset S of a nano topological space $(U, \tau_R(X))$ is said to be a $N\lambda\psi g$ -locally closed set (briefly $N\lambda\psi GLC$ -set) if $S = G \cap H$ where G is $N\lambda\psi g$ -open and H is $N\lambda\psi g$ -closed. The class of all $N\lambda\psi GLC$ sets in $(U, \tau_R(X))$ is denoted by $N\lambda\psi GLC(U, \tau_R(X))$.

Definition 3.2. A subset S of a nano topological space $(U, \tau_R(X))$ is said to be $N\lambda\psi GLC^*$ if there exist a $N\lambda\psi g$ -open set G and a nano closed set H of $(U, \tau_R(X))$ such that $S = G \cap H$. The class of all $N\lambda\psi GLC^*$ sets in $(U, \tau_R(X))$ is denoted by $N\lambda\psi GLC^*(U, \tau_R(X))$.

Definition 3.3. A subset S of a nano topological space $(U, \tau_R(X))$ is said to be $N\lambda\psi GLC^{**}$ if there exist a nano open set G and a $N\lambda\psi g$ -closed set H of $(U, \tau_R(X))$ such that $S = G \cap H$. The class of all $N\lambda\psi GLC^{**}$ sets in $(U, \tau_R(X))$ is denoted by $N\lambda\psi GLC^{**}(U, \tau_R(X))$.

Theorem 3.4. If a subset S of $(U, \tau_R(X))$ is nano locally closed then it is a $N\lambda\psi GLC$ set, $N\lambda\psi GLC^*$ set and $N\lambda\psi GLC^{**}$ set.

Proof. Let S be a nano locally closed subset of U . Then $S = G \cap H$, where G is nano open and H is nano closed in $(U, \tau_R(X))$. By theorem 2.8, S is a $N\lambda\psi GLC$ set, $N\lambda\psi GLC^*$ set and $N\lambda\psi GLC^{**}$ set.

Remark 3.5. The converse of the above theorem need not be true as seen from the following example.

Example 3.6. Let $U = \{a, b, c, d\}$ and with $U/R = \{\{c\}, \{d\}, \{a, b\}\}$ and $X = \{b, c\}$. Then $\tau_R(X) = \{U, \emptyset, \{c\}, \{a, b\}, \{a, b, c\}\}$. Then NLC sets = $\{U, \emptyset, \{c\}, \{d\}, \{a, b\}, \{c, d\}, \{a, b, c\}, \{a, b, d\}\}$, $N\lambda\psi GLC(U, \tau_R(X)) = P(X)$, $N\lambda\psi GLC^*(U, \tau_R(X)) = \{U, \emptyset, \{a\}, \{b\}, \{c\}, \{d\}, \{a, b\}, \{a, c\}, \{b, c\}, \{c, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}\}$ and $N\lambda\psi GLC^{**}(U, \tau_R(X)) = \{U, \emptyset, \{a\}, \{b\}, \{c\}, \{d\}, \{a, b\}, \{a, d\}, \{b, d\}, \{c, d\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}, \{a, b, c\}\}$. Here, $\{a\}$ is $N\lambda\psi GLC$, $N\lambda\psi GLC^*$ and $N\lambda\psi GLC^{**}$ but not NLC .

Theorem 3.7. If a subset S of $(U, \tau_R(X))$ is $N\lambda\psi GLC^*$ then it is a $N\lambda\psi GLC$ set.

Proof. Let S be a $N\lambda\psi GLC^*$ set. Then $S = G \cap H$, where G is an $N\lambda\psi g$ -open set in $(U, \tau_R(X))$ and H is a nano closed set in $(U, \tau_R(X))$. By theorem 2.8, S is $N\lambda\psi GLC$ set.

Remark 3.8. The converse of the above theorem need not be true as shown in the following example.

Example 3.9. Example 3.6, $\{b, d\}$ is $N\lambda\psi GLC$ set but not in $N\lambda\psi GLC^{**}$ set.

Theorem 3.10. If a subset S of $(U, \tau_R(X))$ is $N\lambda\psi GLC^{**}$ then it is a $N\lambda\psi GLC$ set.

Proof. Let S be a $N\lambda\psi GLC^{**}$ set. Then $S = G \cap H$, where G is an nano open set in $(U, \tau_R(X))$ and H is a $N\lambda\psi g$ -closed set in $(U, \tau_R(X))$. By theorem 2.8, S is $N\lambda\psi GLC$ set.

Remark 3.11. The converse of the above theorem need not be true as shown in the following example.

Example 3.12. Example 3.6, $\{b, c\}$ is $N\lambda\psi GLC$ set but not in $N\lambda\psi GLC^{**}$ set.

Theorem 3.13. If $S \in N\lambda\psi GLC(U, \tau_R(X))$ and $T \in N\lambda\psi g$ -closed in $(U, \tau_R(X))$, then $S \cap T \in N\lambda\psi GLC(U, \tau_R(X))$.

Proof. Since $S \in N\lambda\psi GLC(U, \tau_R(X))$, there exist a $N\lambda\psi g$ -open set G and a $N\lambda\psi g$ -closed set H such that $S = G \cap H$. Now $S \cap T = (G \cap H) \cap T = G \cap (H \cap T)$. Since G is $N\lambda\psi g$ -open and $H \cap T$ is $N\lambda\psi g$ -closed, $S \cap T \in N\lambda\psi GLC(U, \tau_R(X))$.

Theorem 3.14. If $S \in N\lambda\psi GLC^*(U, \tau_R(X))$ and T is nano closed in $(U, \tau_R(X))$, $S \cap T \in N\lambda\psi GLC^*(U, \tau_R(X))$.

Proof. Since $S \in N\lambda\psi GLC^*(U, \tau_R(X))$, there exist a $N\lambda\psi g$ -open set G and a nano closed set H such that $S = G \cap H$. Since T is a nano closed set, we have $S \cap T = (G \cap H) \cap T = G \cap (H \cap T)$. Since H is $N\lambda\psi g$ -open and $H \cap T$ is nano closed, $S \cap T \in N\lambda\psi GLC^*(U, \tau_R(X))$.

Theorem 3.15. If $S \in N\lambda\psi GLC^{**}(U, \tau_R(X))$ and T is $N\lambda\psi g$ -closed (resp nano open) in $(U, \tau_R(X))$, then $S \cap T \in N\lambda\psi GLC^{**}(U, \tau_R(X))$.

Proof. Since $S \in N\lambda\psi GLC^{**}(U, \tau_R(X))$, there exist an nano open set G and a $N\lambda\psi g$ -closed set H such that $S = G \cap H$. Now $S \cap T = (G \cap H) \cap T = G \cap (H \cap T)$. Since G is nano open and $H \cap T$ is $N\lambda\psi g$ -closed, $S \cap T \in N\lambda\psi GLC^{**}(U, \tau_R(X))$.

In this case T being a nano open set, we have $S \cap T = (G \cap H) \cap T = (G \cap T) \cap H$. Since $G \cap T$ is open and H is $N\lambda\psi g$ -closed, $S \cap T \in N\lambda\psi GLC^{**}(U, \tau_R(X))$.

4 $N\lambda\psi GLC$ - Continuous Functions

Definition 4.1. A function $f : (U, \tau_R(X)) \rightarrow (V, \tau'_R(Y))$ is said to be $N\lambda\psi GLC$ -continuous (resp. $N\lambda\psi GLC^*$ -continuous, $N\lambda\psi GLC^{**}$ -continuous) if $f^{-1}(G) \in N\lambda\psi GLC(U, \tau_R(X))$ (resp. $f^{-1}(G) \in N\lambda\psi GLC^*(U, \tau_R(X))$, $f^{-1}(G) \in N\lambda\psi GLC^{**}(U, \tau_R(X))$) for each nano closed set G in $(V, \tau'_R(Y))$.

Theorem 4.2. Let $f : (U, \tau_R(X)) \rightarrow (V, \tau'_R(Y))$ be a function. Then we have the following:

- (i) If f is NLC-continuous, then f is $N\lambda\psi GLC$ -continuous, $N\lambda\psi GLC^*$ -continuous and $N\lambda\psi GLC^{**}$ -continuous.
- (ii) If f is $N\lambda\psi GLC^*$ -continuous function, then f is $N\lambda\psi GLC$ -continuous.
- (iii) If f is $N\lambda\psi GLC^{**}$ -continuous function, then f is $N\lambda\psi GLC$ -continuous.

Proof. (i) Suppose that $f : (U, \tau_R(X)) \rightarrow (V, \tau'_R(Y))$ is NLC-continuous. Let G be a nano closed set of U . Then $f^{-1}(G)$ is a nano locally closed set in U . By theorem 3.4, it follows that f is $N\lambda\psi GLC$ -continuous (resp. $N\lambda\psi GLC^*$ -continuous and $N\lambda\psi GLC^{**}$ -continuous).

(ii) Let $f : (U, \tau_R(X)) \rightarrow (V, \tau'_R(Y))$ be a $N\lambda\psi GLC^*$ -continuous function. Let G be a nano closed set of U . Then $f^{-1}(G)$ is $N\lambda\psi GLC^*$ set in U . By theorem 3.7, it follows that f is $N\lambda\psi GLC^*$ -continuous is $N\lambda\psi GLC$ -continuous.

(iii) Let $f : (U, \tau_R(X)) \rightarrow (V, \tau'_R(Y))$ be a $N\lambda\psi GLC^{**}$ -continuous function. Let G be a nano closed set of U . Then $f^{-1}(G)$ is $N\lambda\psi GLC^{**}$ set in U . By theorem 3.10, it follows that f is $N\lambda\psi GLC^{**}$ -continuous is $N\lambda\psi GLC$ -continuous.

Remark 4.3. The converse of the above theorem need not be true as seen from the following examples.

Example 4.4. (i) Let $\{a, b, c, d\}$ with $U/R = \{\{a\}, \{b\}, \{c, d\}\}$ and $X = \{a, d\}$. Then $\tau_R(X) = \{U, \emptyset, \{a\}, \{c, d\}, \{a, c, d\}\}$. Let $V = \{a, b, c, d\}$ with $V/R = \{\{c\}, \{d\}, \{a, b\}\}$ and $Y = \{b, c\}$. Then $\tau'_R(Y) = \{V, \emptyset, \{c\}, \{a, b\}, \{a, b, c\}\}$. Define a function $f : (U, \tau_R(X)) \rightarrow (V, \tau'_R(Y))$ as $f(a) = b$, $f(b) = a$, $f(c) = d$ and $f(d) = c$. Then f is $N\lambda\psi GLC$ -continuous, $N\lambda\psi GLC^*$ -continuous, $N\lambda\psi GLC^{**}$ -continuous but not NLC continuous.

(ii) Let $\{a, b, c, d\}$ with $U/R = \{\{a\}, \{b, c, d\}\}$ and $X = \{a, d\}$. Then $\tau_R(X) = \{U, \emptyset, \{b, c, d\}\}$. Let $V = \{a, b, c, d\}$ with $V/R = \{\{b\}, \{c\}, \{a, d\}\}$ and $Y = \{b, c\}$. Then $\tau'_R(Y) = \{V, \emptyset, \{b\}, \{a, d\}, \{a, b, d\}\}$. Define a function $f : (U, \tau_R(X)) \rightarrow (V, \tau'_R(Y))$ as $f(a) = a$, $f(b) = b$, $f(c) = c$ and $f(d) = d$. Then f is $N\lambda\psi GLC$ -continuous but not $N\lambda\psi GLC^*$ -continuous.

(iii) Let $\{a, b, c, d\}$ with $U/R = \{\{a\}, \{b, c, d\}\}$ and $X = \{b, c\}$. Then $\tau_R(X) = \{U, \emptyset, \{b, c, d\}\}$. Let $V = \{a, b, c, d\}$ with $V/R = \{\{c\}, \{d\}, \{a, b\}\}$ and $Y = \{b, c\}$. Then $\tau'_R(X) = \{V, \emptyset, \{c\}, \{a, b\}, \{a, b, c\}\}$. Define a function $f : (U, \tau_R(X)) \rightarrow (V, \tau'_R(Y))$ as $f(a) = a$, $f(b) = b$, $f(c) = d$ and $f(d) = c$. Then f is $N\lambda\psi GLC$ -continuous but not $N\lambda\psi GLC^{**}$ -continuous.

Theorem 4.7. If $f : (U, \tau_R(X)) \rightarrow (V, \tau'_R(Y))$ and $g : (V, \tau'_R(X)) \rightarrow (W, \tau''_R(Z))$ are any two functions. Then

- (i) $g \circ f$ is $N\lambda\psi GLC$ -continuous if f is $N\lambda\psi GLC$ -continuous and g is nano continuous.
- (ii) $g \circ f$ is $N\lambda\psi GLC^*$ -continuous if f is $N\lambda\psi GLC^*$ -continuous and g is nano continuous.
- (iii) $g \circ f$ is $N\lambda\psi GLC^{**}$ -continuous if f is $N\lambda\psi GLC^{**}$ -continuous and g is nano continuous.

Proof. (i) Let G be a nano closed set in $(W, \tau''_R(Z))$. Since g is nano continuous, $g^{-1}(G)$ is nano closed set in $(V, \tau'_R(X))$. Again, since f is $N\lambda\psi GLC$ -continuous, $f^{-1}(g^{-1}(G))$ is $N\lambda\psi GLC$ in $(U, \tau_R(X))$. Thus $g \circ f$ is $N\lambda\psi GLC$ -continuous function.

(ii) Let G be a nano closed set in $(W, \tau''_R(Z))$. Since g is nano continuous, $g^{-1}(G)$ is nano closed in $(V, \tau'_R(X))$. Since f is $N\lambda\psi GLC^*$ -continuous, $f^{-1}(g^{-1}(G))$ is $N\lambda\psi GLC^*$ in $(U, \tau_R(X))$. Thus $g \circ f$ is $N\lambda\psi GLC^*$ -continuous function.

(iii) Let G be a nano closed set in $(W, \tau''_R(Z))$. Since g is nano continuous, $g^{-1}(G)$ is nano closed in $(V, \tau'_R(Y))$. Since f is $N\lambda\psi GLC^{**}$ -continuous, $f^{-1}(g^{-1}(G))$ is $N\lambda\psi GLC^{**}$ in $(U, \tau_R(X))$. Thus $g \circ f$ is $N\lambda\psi GLC^{**}$ -continuous function.

5 $N\lambda\psi GLC$ - Irresolute Functions

Definition 5.1. A function $f : (U, \tau_R(X)) \rightarrow (V, \tau'_R(Y))$ is said to be $N\lambda\psi GLC$ -irresolute (resp. $N\lambda\psi GLC^*$ -irresolute, $N\lambda\psi GLC^{**}$ -irresolute) if the inverse image of $N\lambda\psi GLC$ set (resp. $N\lambda\psi GLC^*$ set, $N\lambda\psi GLC^{**}$ set) in $(V, \tau'_R(Y))$ is $N\lambda\psi GLC$ set (resp. $N\lambda\psi GLC^*$, $N\lambda\psi GLC^{**}$ set) in $(U, \tau_R(X))$.

Theorem 5.2. If a function $f : (U, \tau_R(X)) \rightarrow (V, \tau'_R(Y))$ is NLC-irresolute, then f is $N\lambda\psi GLC$ -irresolute (resp. $N\lambda\psi GLC^*$ -irresolute and $N\lambda\psi GLC^{**}$ -irresolute).

Proof. Suppose that f is NLC-irresolute. Let G be a nano locally closed set of $(U, \tau_R(X))$. Then $f^{-1}(G)$ is a nano locally closed set in $(U, \tau_R(X))$. By theorem 3.4, it follows that f is $N\lambda\psi GLC$ -irresolute (resp. $N\lambda\psi GLC^*$ -irresolute and $N\lambda\psi GLC^{**}$ -irresolute).

Remark 5.3. The converse of the above theorem need not be true as seen from the following examples.

Example 5.4. Let $\{a, b, c, d\}$ with $U/R = \{\{a\}, \{b, c, d\}\}$ and $X = \{a, c\}$. Then $\tau_R(X) = \{U, \emptyset, \{a\}, \{b, c, d\}\}$. Let $V = \{a, b, c, d\}$ with $V/R' = \{\{a\}, \{b\}, \{c, d\}\}$ and $Y = \{a, d\}$. Then $\tau'_R(Y) = \{V, \emptyset, \{a\}, \{c, d\}, \{a, c, d\}\}$. Define a function $f : (U, \tau_R(X)) \rightarrow (V, \tau'_R(Y))$ as $f(a) = c$, $f(b) = d$, $f(c) = a$ and $f(d) = b$. Then f is $N\lambda\psi GLC$ -irresolute, $N\lambda\psi GLC^*$ -irresolute, $N\lambda\psi GLC^{**}$ -irresolute but not NLC-irresolute.

Theorem 5.5. Let $f : (U, \tau_R(X)) \rightarrow (V, \tau'_R(Y))$ and $g : (V, \tau'_R(Y)) \rightarrow (Z, \tau''_R(Z))$ be any two functions. Then

(i) $g \circ f : (U, \tau_R(X)) \rightarrow (Z, \tau''_R(Z))$ is $N\lambda\psi GLC$ -irresolute if g is $N\lambda\psi GLC$ -irresolute and f is $N\lambda\psi GLC$ -irresolute.

(ii) $g \circ f : (U, \tau_R(X)) \rightarrow (Z, \tau''_R(Z))$ is $N\lambda\psi GLC$ -continuous if g is $N\lambda\psi GLC$ -continuous and f is $N\lambda\psi GLC$ -irresolute.

Proof. (i) Let $G \in N\lambda\psi GLC(W, \tau''_R(Z))$. Since g is $N\lambda\psi GLC$ -irresolute, $g^{-1}(G)$ is $N\lambda\psi GLC$ in $(V, \tau'_R(Y))$. As f is $N\lambda\psi GLC$ -irresolute, $f^{-1}(g^{-1}(G))$ is $N\lambda\psi GLC$ in $(U, \tau_R(X))$. That is $(g \circ f)^{-1}(G) \in N\lambda\psi GLC(U, \tau_R(X))$. Thus $g \circ f$ is $N\lambda\psi GLC$ -irresolute.

(ii) Let G be a nano closed set in $(W, \tau''_R(Z))$. Since g is $N\lambda\psi GLC$ - continuous, $g^{-1}(G)$ is $N\lambda\psi GLC$ in $(V, \tau'_R(Y))$. Again, since f is $N\lambda\psi GLC$ - irresolute, $f^{-1}(g^{-1}(G))$ is $N\lambda\psi GLC$ in $(U, \tau_R(X))$. Thus $g \circ f$ is $N\lambda\psi GLC$ -continuous.

Theorem 5.6. Let $f: (U, \tau_R(X)) \rightarrow (V, \tau'_R(Y))$ and $g: (V, \tau'_R(Y)) \rightarrow (W, \tau''_R(Z))$ be any two functions. Then

- (i) $g \circ f$ is $N\lambda\psi GLC^*$ - irresolute if f and g are $N\lambda\psi GLC^*$ - irresolute.
- (ii) $g \circ f$ is $N\lambda\psi GLC^{**}$ - irresolute if f and g are $N\lambda\psi GLC^{**}$ - irresolute.
- (iii) $g \circ f$ is $N\lambda\psi GLC^*$ - continuous if f is $N\lambda\psi GLC^*$ - irresolute and g is $N\lambda\psi GLC^*$ - continuous.
- (iv) $g \circ f$ is $N\lambda\psi GLC^{**}$ - continuous if f is $N\lambda\psi GLC^{**}$ - irresolute and g is $N\lambda\psi GLC^{**}$ - continuous.

Proof. (i) Let $G \in N\lambda\psi GLC^*(W, \tau''_R(Z))$. Since g is $N\lambda\psi GLC^*$ - irresolute, $g^{-1}(G)$ is $N\lambda\psi GLC^*$ in $(V, \tau'_R(Y))$. As f is $N\lambda\psi GLC^*$ - irresolute, $f^{-1}(g^{-1}(G))$ is $N\lambda\psi GLC^*$ in $(U, \tau_R(X))$. That is $(g \circ f)^{-1}(G) \in N\lambda\psi GLC^*(U, \tau_R(X))$. Thus $g \circ f$ is $N\lambda\psi GLC^*$ -irresolute.

(ii) Let $G \in N\lambda\psi GLC^{**}(W, \tau''_R(Z))$. Since g is $N\lambda\psi GLC^{**}$ - irresolute, $g^{-1}(G)$ is $N\lambda\psi GLC^{**}$ in $(V, \tau'_R(Y))$. As f is $N\lambda\psi GLC^{**}$ - irresolute, $f^{-1}(g^{-1}(G))$ is $N\lambda\psi GLC^{**}$ in $(U, \tau_R(X))$. Hence $g \circ f$ is $N\lambda\psi GLC^{**}$ - irresolute.

(iii) Let G be a nano closed set in $(W, \tau''_R(Z))$. Since g is $N\lambda\psi GLC^*$ - continuous, $g^{-1}(G)$ is $N\lambda\psi GLC^*$ in $(V, \tau'_R(Y))$. Again, since f is $N\lambda\psi GLC^*$ - irresolute, $f^{-1}(g^{-1}(G))$ is $N\lambda\psi GLC^*$ in $(U, \tau_R(X))$. Hence $g \circ f$ is $N\lambda\psi GLC^*$ - continuous.

(iv) Let G be a nano closed set in $(W, \tau''_R(Z))$. Since g is $N\lambda\psi GLC^{**}$ - continuous, $g^{-1}(G)$ is $N\lambda\psi GLC^{**}$ in $(V, \tau'_R(Y))$. Since f is $N\lambda\psi GLC^{**}$ -irresolute, $f^{-1}(g^{-1}(G))$ is $N\lambda\psi GLC^{**}$ in $(U, \tau_R(X))$. Hence $g \circ f$ is $N\lambda\psi GLC^{**}$ - continuous.

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